

UM-TH-94-20

DPNU-94-23

June, 1994

hep-ph/9406399

Nambu-Jona-Lasinio Model vs QCD

Koichi Yamawaki

Department of Physics

Nagoya University

Nagoya, 464-01, Japan

and

V.I. Zakharov¹

The Randall Laboratory of Physics

University of Michigan

Ann Arbor, MI 48109

Abstract

We argue that the extended Nambu-Jona-Lasinio model which introduces four-quark interactions to describe hadron physics at low energies can be confronted with fundamental quantum chromodynamics by means of spectral sum rules. While there arise rather restrictive bounds on the strength of the effective four-quark interaction in the vector channel, introduction of the four-quark interaction in the pseudoscalar channel resolves a long standing puzzle of the sum rules.

¹On leave of absence from Max-Planck Institute for Physics, Munich

More than thirty years ago Nambu and Jona-Lasinio (NJL) proposed a model of superconductivity type as an effective theory of hadrons at low energies [1]. The basic feature of the NJL model is the introduction of effective four-fermion interaction. In terms of quark interactions the extended version of the NJL model Lagrangian is parameterized by two couplings $G_{S,V}$:

$$L_{\text{int}} = \frac{G_S}{2} \left((\bar{q}\lambda^\alpha q)^2 + (\bar{q}i\gamma_5\lambda^\alpha q)^2 \right) - \frac{G_V}{2} \left((\bar{q}\gamma_\mu\lambda^\alpha q)^2 + (\bar{q}\gamma_\mu\gamma_5\lambda^\alpha q)^2 \right), \quad (1)$$

where q are quark fields, λ^α are the Gell-Mann SU(3) matrices in the flavor space and the color indices are suppressed. Since (1) is an effective Lagrangian, loop integrations are allowed up to an ultraviolet cutoff Λ_{UV} which represents one more parameter. With time going on, the original idea [1] has developed into rich and successful phenomenology of hadrons at low energy (for a review see, e.g., Ref.[2]). The modern way to confront the model with the data is to calculate the parameters of pion interactions at low energies [3].

A crucial theoretical problem – common to most of the models with effective Lagrangians – is whether interaction (1) can be derived from QCD. As usual, the effective Lagrangian should share the symmetries of the fundamental one. If one confines oneself to four-quark interaction, then the two terms in (1) are the only ones consistent with the chiral invariance. As for the four-fermionic form itself, it is mostly heuristic, although attempts of its dynamical derivation have also been made, see in particular review in Ref.[4]. Generally speaking, the use of the effective Lagrangian (1) and of perturbative calculations within the fundamental QCD is justified in different kinematical regions. Namely, the language of (nearly) massless quarks and of gluons becomes rigorous at short distances where the running coupling is small, while the effective Lagrangian (1) describes the large-distance, or the low-momentum dynamics.

In this Letter we will attract attention to the possibility of confronting NJL model and fundamental QCD directly, where by fundamental QCD we will understand QCD sum rules [5]. The QCD sum rules exploit perturbative calculations and push them to as low momenta

as possible. It was demonstrated that the sum rules work down to Euclidean momenta $Q^2 \sim 0.5 \text{ GeV}^2$. On the other hand, a fit to the parameters of the NJL model looks as [2]:

$$\Lambda_{UV} \approx 1.25 \text{ GeV}, G_S \approx 5 \text{ GeV}^{-2}, G_V \approx 10 \text{ GeV}^{-2}. \quad (2)$$

The crucial observation is that Λ_{UV} is in fact rather large. As a result there exists so to say moderate Q^2 ,

$$0.5 \text{ GeV}^2 \leq Q_{\text{moderate}}^2 \leq 1.5 \text{ GeV}^2, \quad (3)$$

where the both approaches claim their validity. In this region one can probe the NJL model via the QCD sum rules. The result is that the value of G_V is too big to be consistent with the sum rules. On the other hand, G_S is needed to resolve a long standing puzzle of QCD sum rules [7], that is, the failure of the standard approach [5] in the pseudoscalar channel.

It might worth emphasizing that the $G_{S,V}$ play different roles in building up the hadron phenomenology. Roughly speaking, G_S is needed to generate spontaneous breaking of the chiral symmetry and to explain existence of nearly massless pion in accordance with the original idea [1]. Introduction of G_V allows us to reproduce the vector meson dominance. Our conclusions are that while the latter function of the extended NJL model can hardly be reconciled with the QCD, the effective interaction (1) with (2) in the pseudoscalar channel can be a missing block of the QCD-based phenomenology.

Let us recall the reader the general features of the sum rules [5]. The sum rules in the ρ channel are formulated in terms of $\Pi^\rho(Q^2)$,

$$\Pi^\rho(Q^2)(q_\mu q_\nu - g_{\mu\nu} q^2) = i \int d^4x e^{iqx} \langle 0 | T \{ j_\mu^\rho(x), j_\nu^\rho(0) \} | 0 \rangle, \quad Q^2 \equiv -q^2, \quad (4)$$

where j_μ^ρ is the quark current with quantum numbers of the ρ meson:

$$j_\mu^\rho = \frac{1}{2}(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d).$$

The function $\Pi^\rho(Q^2)$ satisfies once subtracted dispersion relations

$$\Pi^\rho(Q^2) = \frac{Q^2}{\pi} \int \frac{R^{I=1}(s) ds}{s(s+Q^2)}, \quad (5)$$

where $R^{I=1}(s)$ is the ratio of the cross section of e^+e^- annihilation into hadrons with total isospin $I = 1$ to that of annihilation into $\mu^+\mu^-$ pair; in particular, $R^{I=1}(s)$ is contributed by the ρ meson.

The basic idea of the QCD sum rules [5] is to calculate $\Pi^\rho(Q^2)$ at large Q^2 using the perturbative QCD and then extrapolate the result to as low Q^2 as possible. Moreover, it turns out that the advance towards lowest Q^2 is checked by corrections proportional to powers of Q^{-2} . The coefficients in front of the powers of Q^{-2} are related to the quark condensate $\langle 0|\bar{q}q|0\rangle$ and the gluon condensate $\langle 0|G_{\mu\nu}^a G_{\mu\nu}^a|0\rangle$. Numerically, the power corrections set in around $Q^2 \sim m_\rho^2$. More precisely, the correction are relatively small at such Q^2 but blow up fast at lower Q^2 . Moreover the sum rules are most successful once applied to the Borel transform $\Pi^\rho(M^2)$ of $\Pi^\rho(Q^2)$ defined as

$$\Pi^\rho(M^2) \equiv \hat{L}\Pi^\rho(Q^2) = \lim_{n \rightarrow \infty} \frac{1}{(n-1)!} (-1)^n (Q^2)^n \left(\frac{d}{dQ^2}\right)^n \Pi^\rho(Q^2), \quad (6)$$

where the limit is understood in such a way that

$$n \rightarrow \infty, \quad Q^2 \rightarrow \infty, \quad Q^2/n \equiv M^2 \text{ is fixed}, \quad (7)$$

and it is $M^2 \sim m_\rho^2$ rather than $Q^2 \sim m_\rho^2$ that can be reached starting from large M^2 .

In a somewhat simplified form the sum rules read

$$\begin{aligned} & 1 + \left(\frac{0.2 \text{ GeV}^2}{M^2}\right)^2 - \left(\frac{0.3 \text{ GeV}^2}{M^2}\right)^3 + O(M^{-8}) \\ &= \frac{8\pi^2 m_\rho^2}{g_\rho^2 M^2} \exp(-m_\rho^2/M^2) + \frac{2}{3M^2} \int_{s_0}^{\infty} \exp(-s/M^2) R^{I=1}(s) ds, \end{aligned} \quad (8)$$

where M is a variable, g_ρ is the ρ meson coupling related to the e^+e^- width of the ρ meson, $g_\rho^2/4\pi \approx 3$, and the integral on the right-hand side represents contribution of the continuum. For our purposes it is crucial only that even at $M^2 \approx m_\rho^2$ the left-hand side of the sum rules (8) is calculable within the short-distance approach to QCD, i.e., is dominated by the unit corresponding to the bare loop graph. The sum rules agree with the data, or the right-hand side, to within about 10 per cent at $M^2 \sim 0.5 \text{ GeV}^2$.

Now we come to the crucial point of consistency of the effective interaction (1) with the QCD sum rules. Adding the contribution of interaction (1) to the bare quark loop, we get a correction:

$$\frac{\delta\Pi^\rho(Q^2)}{\Pi_0^\rho(Q^2)} \approx -\frac{G_V Q^2}{2\pi^2} \ln Q^2, \quad (9)$$

or

$$\frac{\delta\Pi^\rho(M^2)}{\Pi_0^\rho(M^2)} \approx \frac{G_V M^2}{\pi^2} (1 - \gamma + \ln M^2), \quad (10)$$

where Π_0^ρ is the contribution of the bare loop with massless quarks and $\gamma \approx 0.577$ is the Euler constant. Note that the effect of the new interaction grows with M^2 . On other other hand, at large M^2 it should disappear because of the onset of the asymptotic freedom. This emphasizes once more that the effective interaction (1) can be valid only at relatively low momenta. Once M^2 approaches Λ_{UV}^2 , we have to allow for a form factor due to the softening of the effective interaction. However, at $M^2 \sim 0.5 \text{ GeV}^2$ the correction (10) is estimated reliably.

To be consistent with the sum rules the correction (10) is to be small at $M^2 \sim 0.5 \text{ GeV}^2$. However, if we substitute the numbers, then we find in fact $\delta\Pi^\rho(0.5 \text{ GeV}^2) \sim \Pi_0^\rho(0.5 \text{ GeV}^2)$. Thus the effect of the new interaction cannot be even treated as a correction and should be iterated. Thus, we conclude that either the value of G_V or of the cutoff Λ_{UV} given by (2) is too high. We will argue next that the consideration of the sum rules in the pion channel favors the former possibility.

Now, we consider sum rules in the π channel in exactly the same way as in the ρ channel outlined above. The corresponding current is defined as

$$j^\pi = \frac{1}{2}(\bar{u}i\gamma_5 u - \bar{d}i\gamma_5 d), \quad (11)$$

and we introduce Π^π in terms of the correlator similar to (4) but without factorizing ($q^2 g_{\mu\nu} - q_\mu q_\nu$). The sum rules take the form [7]:

$$\left(\frac{\alpha_s(M^2)}{\alpha_s(\mu^2)}\right)^{8/9} \left(1 + \left(\frac{0.3 \text{ GeV}^2}{M^2}\right)^2 + 2\left(\frac{0.2 \text{ GeV}^2}{M^2}\right)^3 + O(M^{-8})\right) \quad (12)$$

$$= \frac{16\pi}{3M^4} \int ds \exp(-s/M^2) \text{Im}\Pi^\pi(s),$$

where $\text{Im}\Pi^\pi(s)$ is the imaginary part of the correlator of two currents j^π , the factor $(\alpha_s(M^2)/\alpha_s(\mu^2))^{8/9}$ is due to a non-vanishing anomalous dimension of j^π and α_s is the running QCD coupling. As far as $\text{Im}\Pi^\pi(s)$ is concerned, the only well-known contribution to it comes from the pion:

$$\text{Im}\Pi_{\text{pole}}^\pi = \pi f_\pi^2 m_\pi^4 (m_u + m_d)^{-2} \delta(s - m_\pi^2), \quad (13)$$

where $f_\pi \approx 93$ MeV and $m_{u,d}$ are the current quark masses.

Now, it has been demonstrated that the sum rules (12) do not hold experimentally as they are stated [7]. The point is that the pole contribution (13) alone, with negligence of the rest of $\text{Im}\Pi^\pi(s)$ which is positive definite, is too large to be consistent with (12). More specifically, one can prove existence of a new contribution, unaccounted in the sum rules, for M^2 ranged between $0.5 \text{ GeV}^2 < M^2 < 2 \text{ GeV}^2$. At $M^2 \sim 0.5 \text{ GeV}^2$ the new contribution is no less than that of the pion, while at $M^2 \sim 2 \text{ GeV}^2$ it is still larger than 10 per cent of the pion contribution. The sensitivity of the sum rules at lower M^2 is limited by the power corrections and at larger M^2 the bare loop may dominate.

What we propose here is to ascribe this new contribution to the effective interaction (1) in the π channel. Consider first the G_S term as a perturbation. Then

$$\frac{\delta\Pi^\pi(Q^2)}{\Pi_0^\pi(Q^2)} \approx \frac{3G_S Q^2}{4\pi^2} \ln Q^2, \quad (14)$$

or

$$\frac{\delta\Pi^\pi(M^2)}{\Pi_0^\pi(M^2)} \approx -\frac{3G_S M^2}{\pi^2} \left(\frac{3}{2} - \gamma + \ln M^2 \right). \quad (15)$$

This equation is supposed to be valid at M^2 much smaller than Λ_{UV}^2 . Applying it at $M^2 = 0.5 \text{ GeV}^2$, we find:

$$\frac{\delta\Pi^\pi(M^2 = 0.5\text{GeV}^2)}{\Pi_0^\pi(M^2 = 0.5\text{GeV}^2)} \sim 2.5. \quad (16)$$

Literally, the analysis of the sum rules suggests a new contribution of order one, in the same units. The factor we get now is in rough agreement with this estimate.

To get a better estimate we should have iterated the effect of the new interaction, since it turns out to be large. The change brought by the iterations of the effective interaction within the NJL model is remarkably simple and well known. Namely, within the the NJL model the summation of the loops generated by the G_S term produces a pion. In this way one reproduces the contribution of the pion into the right-hand side of eq.(12) and explains the failure of the sum rules which do not account for the interaction (1).

Thus the new contribution to the sum rules gets a natural explanation. Namely, the four-fermion interaction gives rise to the pion as proposed in the original papers [1]. At $M^2 \sim \Lambda_{UV}^2 \sim 1.5 \text{ GeV}^2$ the effective interaction is dissolved and the sum rules get dominated by the bare quark loop graph. The estimate of the mass scale of the onset of asymptotic freedom in the pion channel as 2 GeV^2 (see above) turns out to be in reasonable agreement with the estimate of Λ_{UV}^2 within the NJL model.

This picture by itself does not explain the difference between the vector and the pseudoscalar channels. There should be a new kind of correction to the sum rules [7] within the fundamental QCD. If the new correction is a $1/M^2$ term [6] associated with the ultraviolet renormalon, then the difference between the channels is not so dramatic, say, a factor of 4 would suffice. This interpretation is also supported by the recent observation that ultraviolet renormalon is associated with four-quark interactions [8]. However, there is no reliable way to find the relative weight of G_S and G_V .

The chain of the arguments get closed through the prediction obtained above,

$$G_V \ll G_S, \tag{17}$$

which can be tested within the NJL model. Inspection of the most recent fits [3] reveals independent evidence in favor of (17). In particular, any $G_V \neq 0$ drives the predicted value of the constant g_A governing the beta-decay of neutron off its experimental value. Less dramatically, taking $G_V = 0$ improves agreement with the data in some other cases as well. Furthermore, as is noted in [3] the NJL model with $G_V = 0$ is equivalent to the

effective QCD Lagrangian of Ref.[9] which describes successfully nonleptonic weak decays and electromagnetic properties of the pion [10].

Thus, it seems fair to say that the NJI model with $G_V = 0$, $G_S \neq 0$ results in a sound phenomenology, although it puts π and ρ mesons on different footing.

The authors are thankful to G. Grunberg and M.K. Volkov for useful discussions. One of the authors (V.I.Z.) acknowledges warm hospitality extended to him during his stay with the theory group at Nagoya University when this work was started. Part of this work was done while K.Y. was at Institute for Theoretical Physics, University of California at Santa Barbara (Workshop "Weak Interactions"). The work was supported in part by the U.S. Department of Energy, and by a Grant-in-Aid for Scientific Research from the Japanese Ministry of Education, Science and Culture (No. 05640339) and the Ishida Foundation.

References

- [1] Y. Nambu and G. Jona-Lasinio, *Phys. Rev.* **122** (1961) 345; **124** (1961) 246.
- [2] M.K. Volkov, *Ann. Phys.* **157** (1984) 282.
- [3] J. Bijnens, C. Bruno and E. de Rafael, *Nucl. Phys.* **B390** (1993) 501.
- [4] R.D. Ball, *in* Proceedings of the Workshop on Skyrmions and Anomalies, Mogilany, Poland, (1989), p.54.
- [5] M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, *Nucl.Phys.* **B147** (1979) 385; 448; L.J. Reinders, H.R. Rubinstein and S. Yazaki, *Phys.Repts.* **127** (1985) 1.
- [6] V.I. Zakharov, *Nucl.Phys.* **B385** (1992) 452.
- [7] V.A. Novikov, M.A. Shifman, A.I. Vainshtein, and V.I. Zakharov, *Nucl. Phys.* **B191** (1981) 301.

- [8] A.I. Vainshtein and V.I. Zakharov, preprint TPI-MINN-94/9-T, UM-TH-94-09, April 1994.
- [9] D. Espriu, E. de Rafael and J. Taron, *Nucl. Phys.* **B345** (1990) 22.
- [10] A. Pich and E. de Rafael, *Nucl. Phys.* **B358** (1991) 311; J. Bijnens, *Nucl. Phys.* **B367** (1991) 709; J. Bijnens and E. de Rafael, *Phys. Lett.* **B273** (1991) 483.